https://doi.org/10.24867/GRID-2022-p70 Original scientific paper

# CALCULATION OF CUTTING FORCE BY BOOK-EDGE TRIMMING WITH DISK KNIVES

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Abstract*: The paper proposes formulas for theoretical calculation of forces when trimming book-edges with a disc knife. The theoretical model is based on experimental data obtained by the authors. The authors calculated the approximation parameters from own experimental studies. The kinematic parameters of the cutting process and the approximated value of the unit cutting force were used in the calculation formulas. Calculation makes it possible to predict value of total cutting force depending on diameter and sharpening angle of knife, value and direction of its angular speed, feed rate, thickness and distance of book block from axis of knife rotation.*

Key words: book cutting, circular knife, disk knives, book-edge trimming

## 1. PREFACE

Paper cutting operations are important operations of printing. The process of cutting paper stacks and books with a circular knife is relatively poorly researched. Krabisch (1962) found that the strength and quality of the trimming of brochures largely depend on the thickness of the block, the feed rate, and the rotary speed and direction of rotation of the circular knife. The main obstacle in using this method for processing thick book blocks is a significant increase in trimming forces and the phenomenon of excessive heating of the knife as a result of rubbing against the paper, which causes local "burns" and deterioration of the cutting quality. Research conducted by Grushevski (1963) confirmed the limitation of the possibility of qualitative trimming of books. Anyway, some of known cutters are used for book-edge trimming and brochure cutting. In order to speed up the process and to select its optimal parameters some additional researches needed.

## 2. BACKGROUND

Various authors studied the cutting of various materials with disk knives, while various hypotheses about the direction of the cutting forces, about the effect of the transformation of the knife-sharpening angle and about the ratio of the forces of normal cutting and sawing were proposed. As for the direction of the cutting force, two main hypotheses were considered, Daurski & Machihin (1980): the total cutting force is directed against the speed, or the cutting force has two components: the normal force, directed normally to the knife blade, and the sawing force, directed tangentially to the blade. According to the results of our experiments, the first hypothesis is not confirmed when cutting book blocks, since the vector of total cutting force is always directed at some angle to the vector of cutting speed. When applied to a flat knife moving along an inclined trajectory, Reznik (1975) suggests introducing a sliding cutting coefficient equal to the ratio of horizontal force to vertical force, i.e. the ratio of sawing force to cutting force. Obviously, in this case, the sawing force also includes the friction force during the sliding movement of the knife along the book block. In previous papers, the authors have dealt with theoretical studies of the kinematics of the process of cutting book blocks with a circular knife mounted centrally and eccentrically, and published some results of experimental studies of cutting forces, (Janicki, 2020; Janicki, Petriaszwili & Komarov, 2016; Janicki, Petriaszwili & Komarov, 2017; Petriaszwili, Janicki & Komarov, 2019; Petriaszwili, Janicki & Komarov, 2020; Petriaszwili, Janicki & Komarov, 2021; Petriaszwili & Janicki, 2017). This paper attempts to relate the results of theory and experiment and to establish some mathematical relationships between the kinematic parameters of the process and the cutting forces.

## 3. EXPERIMENTAL RESEARCH

Our testing equipment allowed us to register the change in the longitudinal force *Fx*, transverse force *Fz* and vertical force *Fy*. The studies conducted on the equipment, showed that the value of the vertical component of the cutting force ranges a few percent of the values of longitudinal and transverse forces so it can be ignored (Petriaszwili, Janicki & Komarov, 2021). Figure 1 shows the relationship of the recorded forces *Fx* and *Fz* with the normal cutting force *Fn* and the tangential force, or sawing force, *Ft*. Knowing the values of the registered components of the total cutting force *F*, it is easy to calculate the appropriate values of the normal force and the sawing force (Figure 1).

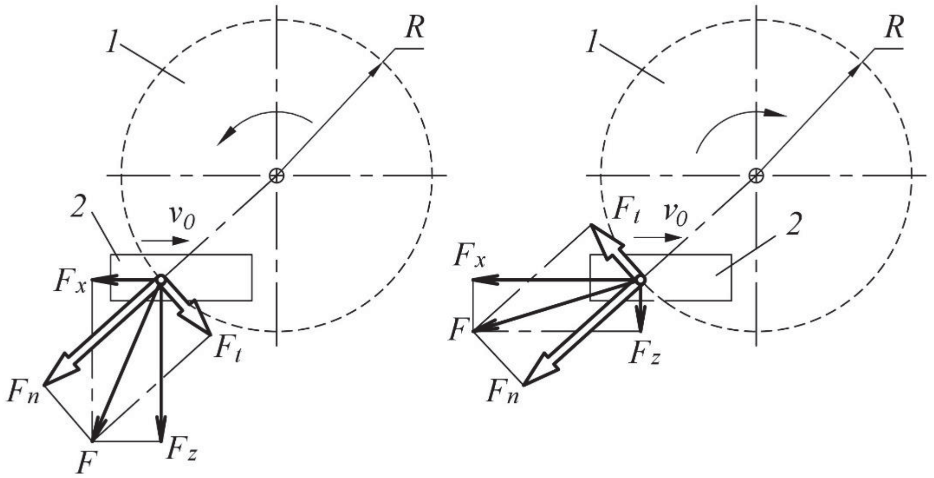


*Figure 1: Different representation of the components of the forces acting on the block from the side of the knife during the forward cutting (left) and reverse cutting (right).*
*F - total cutting force, 1 - circular knife, 2 - book block.*

Figure 2 shows typical graphs of dependence of the measured cutting forces *Fx* and *Fz* on the distance to the knife rotation axis. Graphs of normal *Fn* and tangential force *Ft* in the right chart are obtained by means of recalculations according to Figure 1.

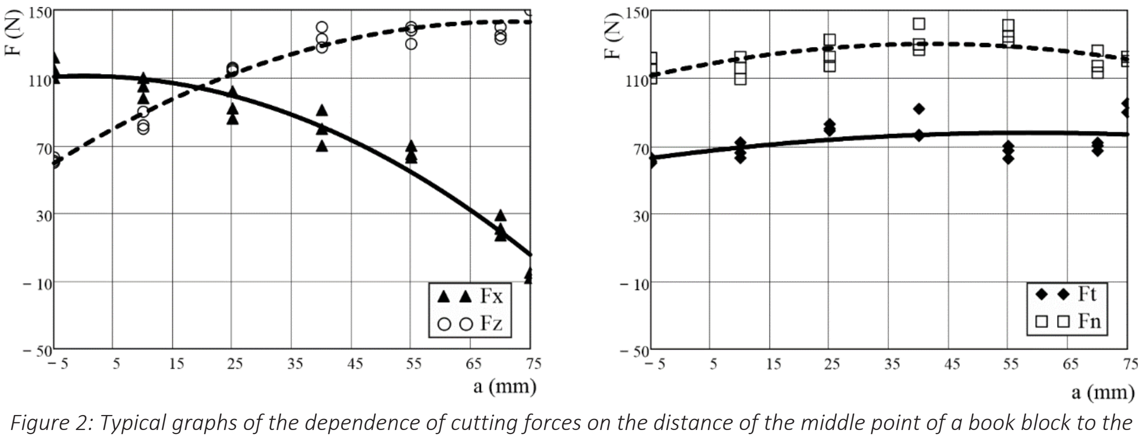


*Figure 2: Typical graphs of the dependence of cutting forces on the distance of the middle point of a book block to the axis of rotation of the knife. Left – measured forces, right – recalculated nomal and tangential forces*

The result of our experiments showed that the ratio between the tangential force and the normal force, or the sliding cutting coefficient, is an almost constant value for certain conditions. This is evident from the right chart in the Figure 2. Hence, for further research it is sufficient to determine what the normal cutting force depends on. To do this, it is convenient to represent it in the form of unit cutting force, which is equal to the ratio of a value of the normal force to the length of the cut. Unlike a flat knife, the cutting length of a book block with a circular knife depends on the height of the book block and its position relative to the knife rotation axis (Figure 3).

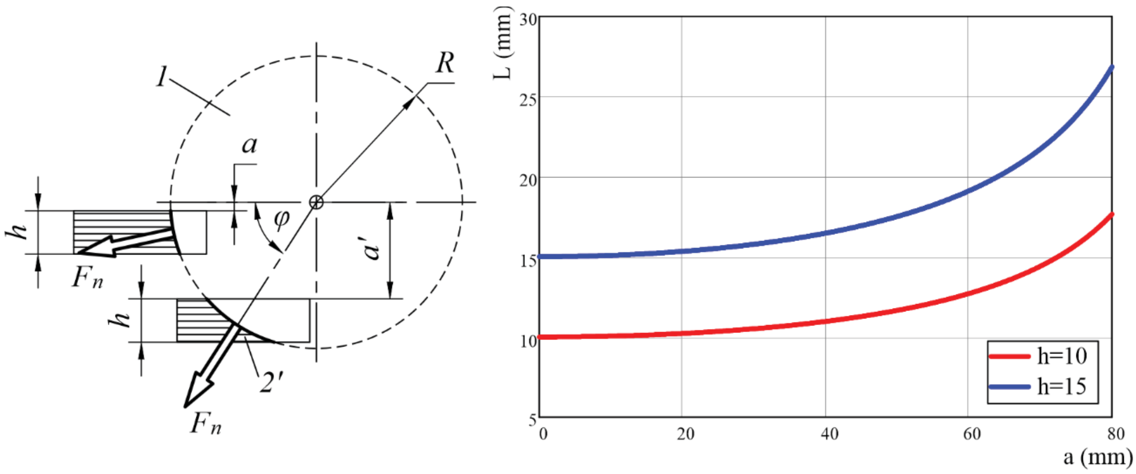


*Figure 3: To the calculation of the arc cut length L. 1 - circular knife, 2 - book block, h - block thickness, a - distance to block from the knife rotation axis, R - knife radius. Left – two different positions of a book block, right – cut length depending on the distance a. Knife radius R=98 mm, block height h= 10 mm and 15 mm.*

It is known from the studies by Mordovin (1962), that the main factor influencing the value of the unit normal force of cutting into a pile of paper or into a book block is the value of the transformed angle of blade sharpening. For a rotating circular knife, this angle is determined by the formula (Janicki & Petriaszwili, 2015):

$$\alpha_T = \arctan\left[\tan(\alpha_0)\frac{v_0\cos(\varphi)}{\sqrt{[v_R\sin(\varphi)\pm v_0]^2+[v_R\cos(\varphi)]^2}}\right] \qquad (1)$$

Where the "minus" sign should be taken for the forward cutting and the "plus" sign for the reverse cutting. The value of the transformed angle depends on the position of the cutting point on the disk knife blade, i.e. on the angle $\varphi$, the feed rate $v_0$, the linear velocity of the blade at the cutting point $v_R$ and the sharpening angle of the knife blade $\alpha_0$. It follows from Figure 3 that the block position affects not only the cutting length, but also the direction of the normal force and its magnitude associated with the transformed sharpening angle. Figure 4 illustrates the change in the transformed blade sharpening angle at the midpoint of the cut as a function of the distance from the axis of rotation.

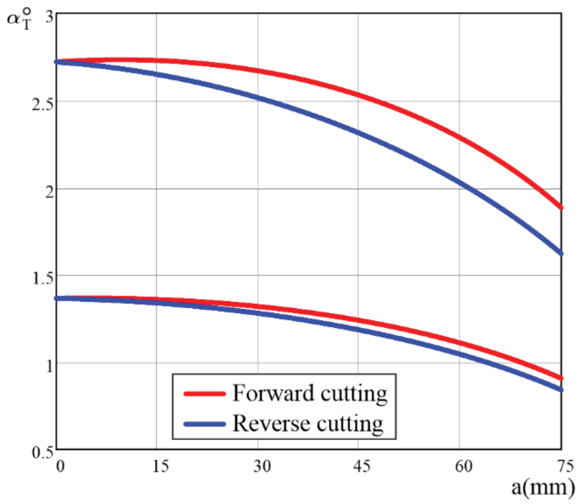


*Figure 4: Transformed angle $\alpha_T$ depending on the distance of a book block from the knife rotation axis a. Cutting parameters: R = 98 mm, $\alpha_0$ = 26 °, $v_o$ = 0,5 m/sec, h = 10 mm, top curves - n = 500 rpm, bottom curves - n = 1000 rpm*

As the distance to the axis of rotation increases, the transformed angle decreases, hence the specific normal force should decrease. But the cutting length increases, which in turn should increase the total normal force. As a result, the value of normal cutting force remains practically unchanged, which can be seen from Figure 2. According to the research of some authors, Ginzburg (1957), Komarov & Petriaszwili (1989), the normal cutting force with a flat knife can be approximated with sufficient accuracy by a power dependence in the form:

$$F = K_0 \cdot L \cdot \alpha_T^{\gamma} \qquad (2)$$

Where $\alpha_T$ - transformed sharpening angle, $L$ – cut length, $K_0$ and $\gamma$ – empiric coefficients. Let us define the unit cutting force $q_n$ as the normal force $F_n$ divided by the length of the cut $L$. The empirical coefficients depend primarily on the type of paper being cut. We apply this approach to the approximation of the normal cutting force for a circular knife. When cutting with a circular knife, unlike a flat knife, the cut is arc-shaped, and the transformed cutting angle changes along this arc. But the thickness of the block is small compared to the diameter of the knife, so the arc of cut can be treated as a chord and the transformed angle - as a constant. For approximation, as follows from formula (2), we need to know the dependence of the unit force on the transformed cutting angle. We obtained the experimental dependences of the cutting force on the block distance to the knife rotation axis (see Figure 2). Let us calculate the value of the transformed sharpening angle and the cutting length for each measured point. Knowing the value of the cutting length, we calculate the unit normal force, take the transformed sharpening angle as the argument, and plot the corresponding dependencies. Figure 5 shows such dependences calculated from the data shown in Figure 2.

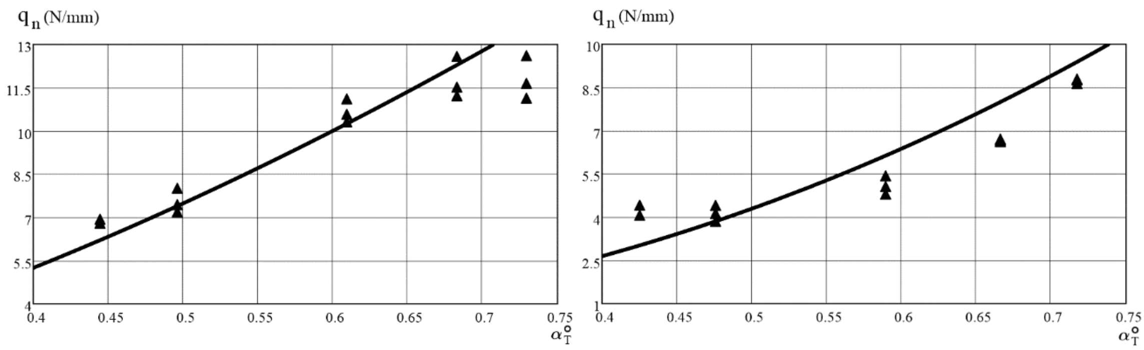


*Figure 5: Dependencies of normal unit force $q_n$ on transformed sharpening angle. Cutting parameters: diameter of the knife D=196mm, sharpening angle $\alpha_0$ = 26 °, feed rate $v_o$ = 0,5 m/sec, knife speed n=1800rpm, block thickness h=10mm, offset paper 70 gsm, left- forward cutting, right- reverse cutting*

Regression analysis and approximation by formula (2) were performed by the method of least squares in Mathcad. The approximated values of the empirical coefficients are somewhat different for the forward and reverse cutting and for different feed rates and knife rotation speeds. But this is not essential for understanding of general regularities. In our estimation, the coefficients can take the following values: $K_0$=19.3 … 22.6, $\gamma$=1.6 … 2.2. According to our experiments, the ratio between the tangential force and the normal force remains close to constant and depends on the thickness of the block. As an example, Figure 6 shows the relationship between the tangential and normal forces at different positions of the block relative to the rotation axis of the knife.

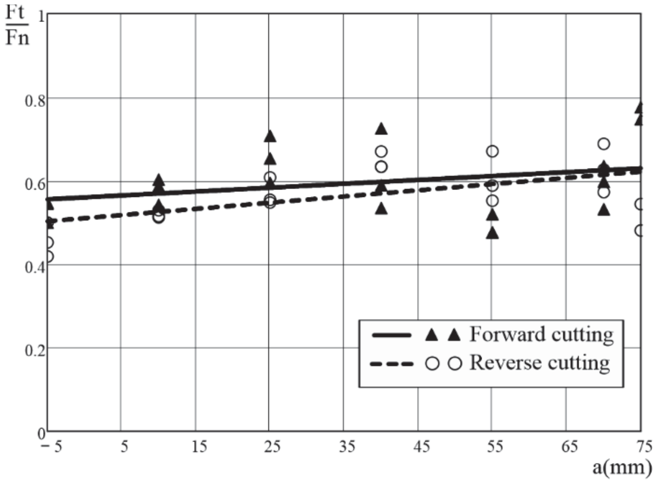


*Figure 6: Ratio of tangential and normal forces. Cutting parameters are the same as in Figure 5*

## 4. THEORETICAL CALCULATION OF FORCES

Based on the above, we will make the following assumptions when deriving formulas:

- The book block remains stationary while the knife rotates and moves toward the block at feed rate.
- The total cutting force acting from the blade on the block consists of a normal component, directed along the radius, and a tangential component, directed tangentially to the blade in the direction of the linear velocity of the blade point. The normal component raises due to the destruction of the sheets of paper as the blade plunges in, and the tangential component is equal to the sum of the sawing force and the frictional force.
- The value of the unit normal component of the cutting force depends on the value of the transformed angle of the knife blade sharpening at the cutting point according to formula (2).
- The sliding cutting coefficient is equal to the ratio of the tangential component to the normal component of the cutting force, and is a constant for certain cutting conditions.

Let us imagine a disk knife blade as a set of elementary straight prismatic knives of infinitely small length dl with the sharpening angle corresponding to the transformed angle in the current position of the elementary knife on the blade. Each elementary knife moves along its own trajectory (Figure7.):

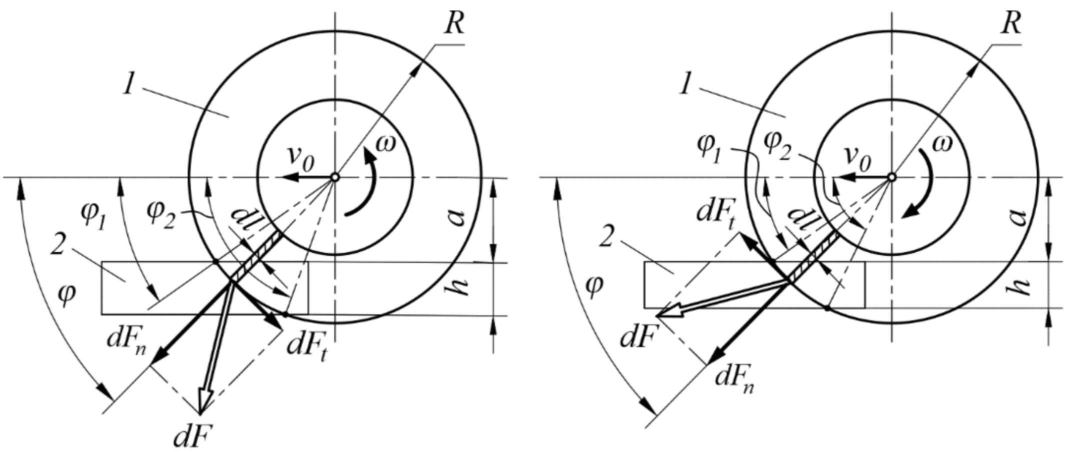


*Figure 7: Diagrams for calculating the cutting forces acting on the book block.*
*1 - circular knife, 2 - book block.Left- forward cutting, right- reverse cutting, $v_0$- block feed rate,$v_R$ - linear speed of knife points on the blade, φ- angle of the knife rotation, ω- angular speed of the knife, h - thickness of the book block, a - distance of the block from the knife rotation axis*

The elementary knife performs oblique-sliding cutting. Let us calculate the elementary normal cutting force $dF_n$ on the blade of the elementary knife taking into account (1) and (2):

$$dF_n = K_0 \left\{ \arctan\left( \tan(\alpha_0) \frac{v_0 \cdot \cos(\varphi)}{\sqrt{[v_R \sin(\varphi) - v_0]^2 + [v_R \cos(\varphi)]^2}} \right) \right\}^{\gamma} dl \tag{3}$$

We express the elementary tangential force through the sliding cutting coefficient *f,* which we assume to be constant:

$$dF_t = dF_n \cdot f \tag{4}$$

To find the total longitudinal and transverse cutting forces, we need to project the normal and tangential components onto the horizontal and vertical axes and integrate the obtained expressions with the angle $\varphi$ from $\varphi_1$ up to $\varphi_2$. Let us perform the obvious transformations and obtain the final dependences:

$$F_x = \int_{\arcsin\left(\frac{a}{R}\right)}^{\arcsin\left(\frac{a+h}{R}\right)} K_0 \left\{ \arctan\left[ \tan(\alpha_0) \frac{v_0 \cos(\varphi)}{\sqrt{[\omega R \sin(\varphi) \pm v_0]^2 + [\omega R \cos(\varphi)]^2}} \right] \right\}^{\gamma} R\left[\cos(\varphi) \pm f \cdot \sin(\varphi)\right] d\varphi$$

(5)

$$F_z = \int_{\arcsin\left(\frac{a}{R}\right)}^{\arcsin\left(\frac{a+h}{R}\right)} K_0 \left\{ \arctan\left[ \tan(\alpha_0) \frac{v_0 \cos(\varphi)}{\sqrt{[\omega R \sin(\varphi) \pm v_0]^2 + [\omega R \cos(\varphi)]^2}} \right] \right\}^{\gamma} R\left[\sin(\varphi) \mp f \cdot \cos(\varphi)\right] d\varphi$$

The upper signs are taken for reverse cutting, the lower signs - for the forward cutting. Integral in formulas (5) cannot be taken in closed form, that is why we carried out calculation by numerical methods using Mathcad. Comparison with experimental data shows, that formulas (5) give a good approximation in a certain range of parameters. As an example, Figure 8-10 show a comparison of experimental data and calculations by formulas (5) for different cutting parameters.

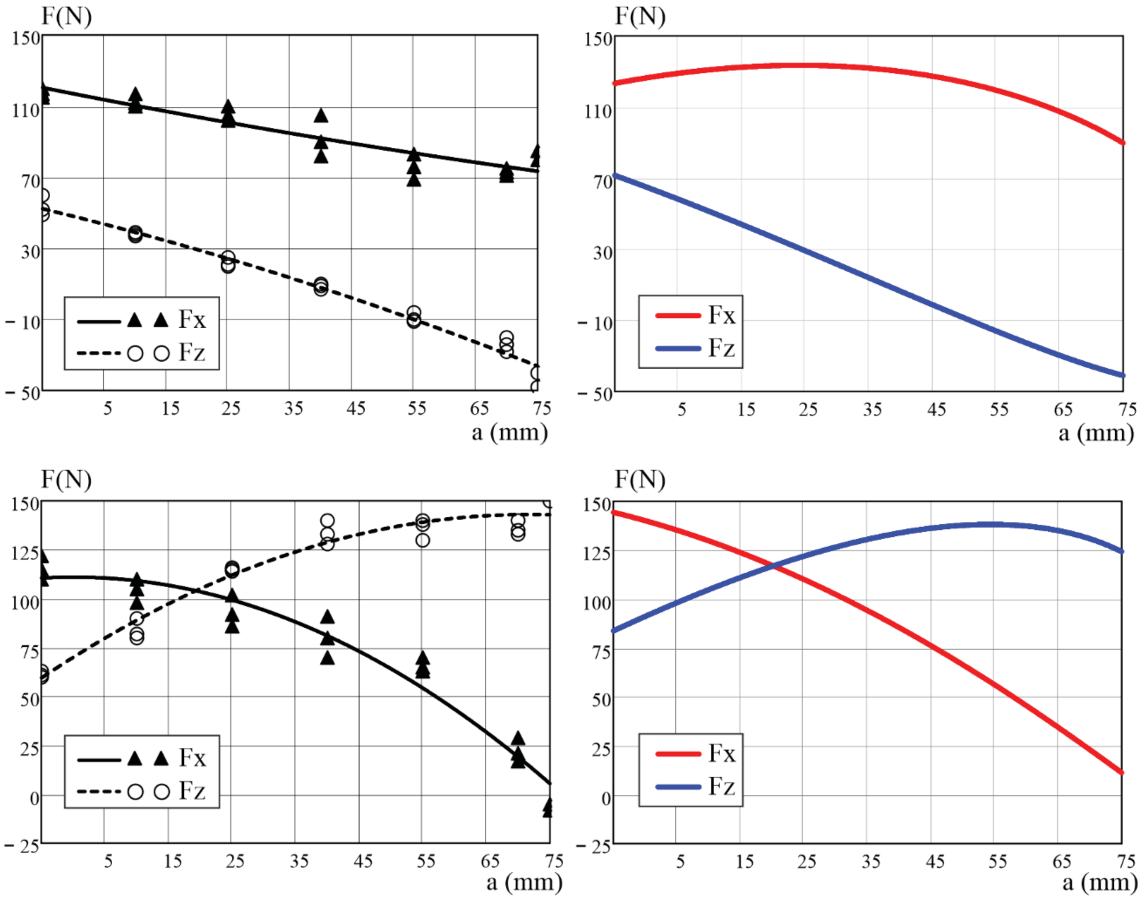


*Figure 8: Comparison of the values of longitudinal and transverse forces at different distances of the block from the axis of rotation of the blade. The upper drawings represent the reverse cutting, the lower ones show the forward one. On the left there are results of the experiment, on the right - calculations according to formulas (5)*

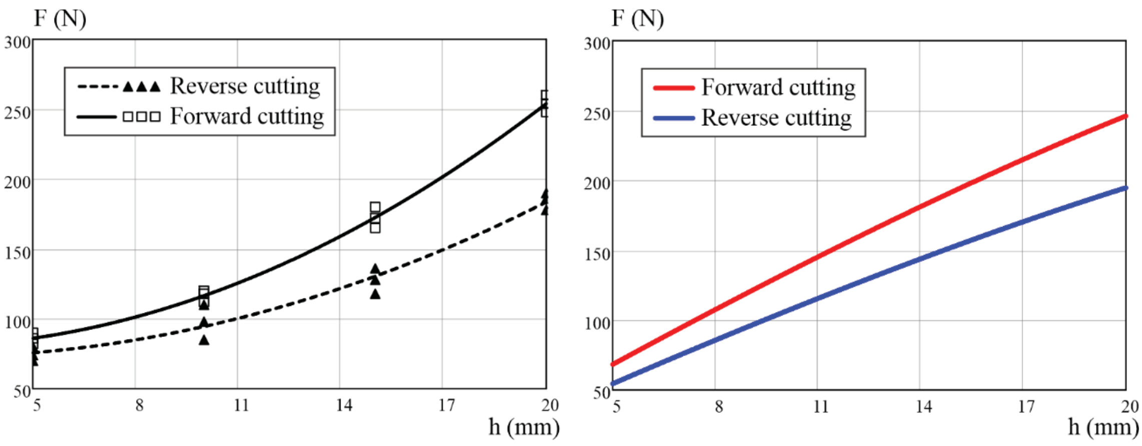


*Figure 9: Comparison of the dependence of the total cutting force on the block thickness. On the left there are results of the experiment, on the right - calculations according to the formulas (5)*

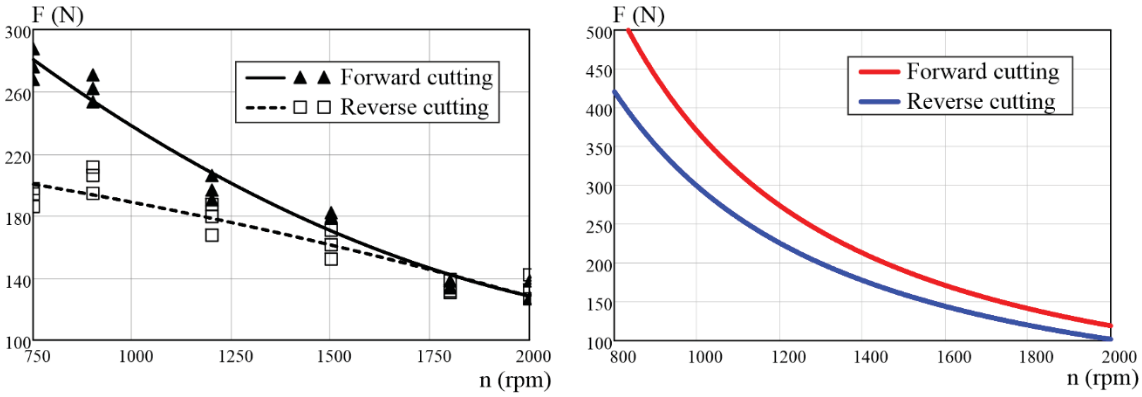


*Figure 10: Comparison of the dependence of the total cutting force on the knife angular speed.*
*On the left there are results of the experiment, on the right - calculations according to the formulas (5)*

Calculated charts of cutting forces somewhat differ from the charts built according to the measured forces. They differ by value and by form due to the simplified model. But they show trends and common dependencies on different parameters of cuttings. The largest deviation is demonstrated by the dependence on the knife speed at relatively low rpm (Figure 10). However, in the range of operating speeds, the approximation accuracy is quite acceptable. The other dependences show a good approximation.

## 5. CONCLUSIONS

Comparison of the calculated values of the cutting force with the experimental ones shows a good match of the results in a certain range of parameters. With the help of these formulas, it is possible to study the influence of various settings for the process of cutting a book block with a disk knife with sufficient accuracy. There is no need to perform time-consuming and expensive experimental studies in full.

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